Session 7: Lazy evaluation

COMP2221: Functional programming

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Recap

- Saw `type` and `data` declarations
- Discussed difference between sum and product types
- Saw some more on type classes
- `Functor` as a type class for mappable containers
- *Functor laws*
  - `fmap id == id`
  - `fmap (f . g) == fmap f . fmap g`
  - How to prove this for a datatype (inductively, or by exhaustive enumeration).
- Discussed why one might want to implement type class instances for our data types
- Saw how `data` declarations allowed for recursive types ⇒ *infinite data structures*

Lazy evaluation

How does this work?

Fibonacci sequence

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

```haskell
fibs = 0 : 1 : zipWith (+) fibs (tail fibs)
Prelude> take 10 fibs
[0,1,1,2,3,5,8,13,21,34]
```

How long?

```python
def slow_function(a):
    ... # 5 minute computation

def compute(a, b):
    if a == 0:
        return 1
    else:
        return b

compute(0, slow_function(0))
compute(1, slow_function(1))
```

```haskell
slow_function :: Int -> Int
-- 5 minute computation
slow_function a = ...

compute :: Int -> Int -> Int
compute a b | a == 0    = 1
            | otherwise = b

compute 0 (slow_function 0)
compute 1 (slow_function 1)
```

Lazy evaluation: AKA I'll get it when you ask

- Not only is Haskell a pure *functional* language
- It is also evaluated *lazily*
- Hence, we can work with infinite data structures
- …and defer computation until such time as it's strictly necessary

Definition (Lazy evaluation)

Expressions are not evaluated when they are bound to variables. Instead, their evaluation is *deferred* until their result is needed by other computations.

Evaluation strategies

- Haskell's basic method of computation is *application* of functions to arguments
- Even here, though we already have some freedom

Example

```
inc :: Int -> Int
inc n = n + 1

inc (2*3)
```

Two options for the evaluation order

→ reduce expression

```
inc (2*3)
= inc 6 -- applying *
= 6 + 1 -- applying inc
= 7 -- applying +
```

```
inc (2*3)
= (2*3) + 1 -- applying inc
= 6 + 1 -- applying *
= 7 -- applying +
```

- As long as all the expression evaluations *terminate*, the order we choose to do things doesn't matter. → in terms of correctness

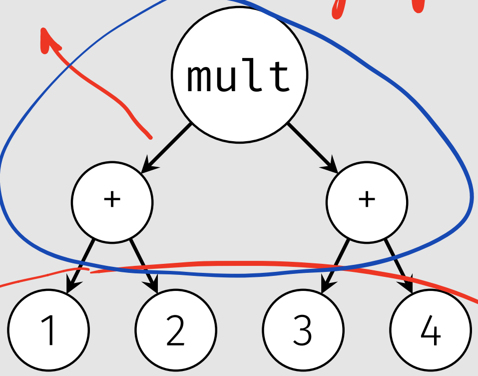
Evaluation strategies II

- We can represent a function call and its arguments in Haskell as a graph
- Nodes in the graph are either *terminal* or *compound*. The latter are called *reducible expressions* or *redexes*

Example

```haskell
mult :: (Int, Int) -> Int
mult (x, y) = x*y

mult (1+2, 3+4)
```



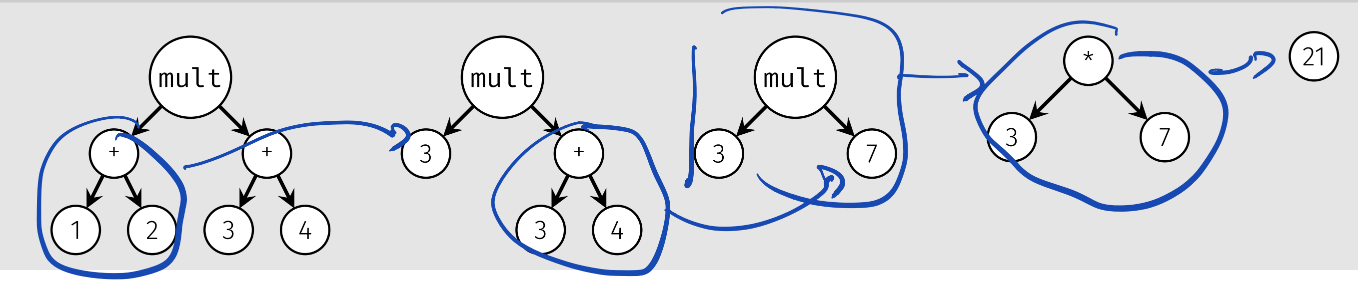
- 1, 2, 3, and 4 are terminal (not reducible) expressions
- (+) and `mult` are reducible expressions.

Innermost evaluation

expression graph

- Evaluate "bottom up"
- First evaluate redexes that only contain terminal or *irreducible* expressions, then repeat
- Need to specify evaluation order at leaves. Typically: "left to right"

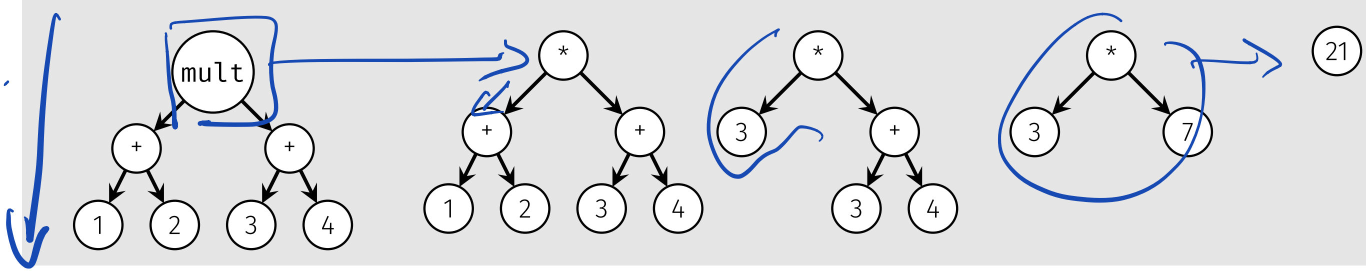
Example



Outermost evaluation

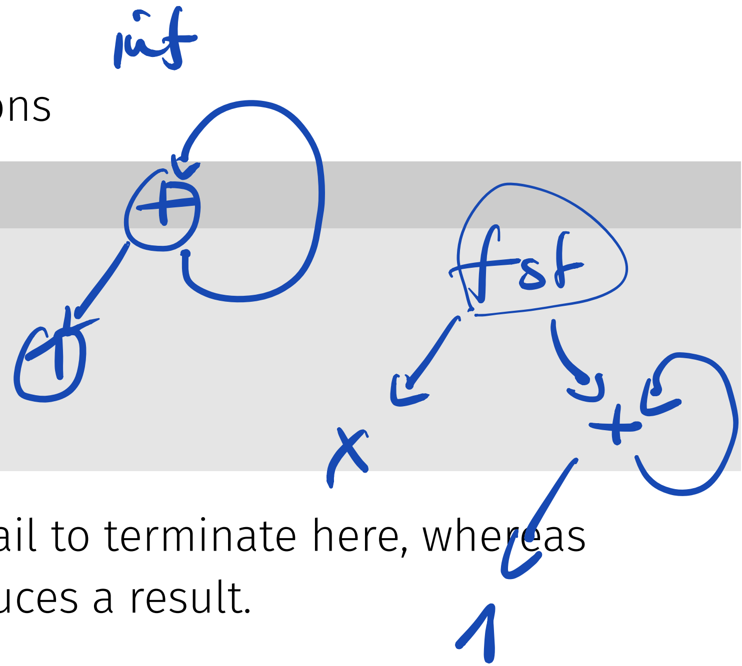
- Evaluate "top down"
- First evaluate redexes that are outermost, then repeat
- Again, need an evaluation order for children, typically choose "left to right".

Example



Termination

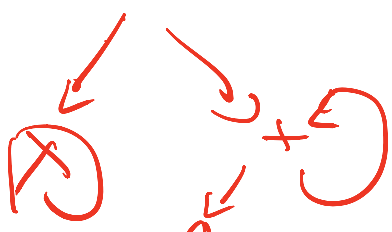
- For *finite* expressions, both innermost and outermost evaluation terminate.
- Not so for infinite expressions



Example

```
inf :: Integer
inf = 1 + inf
fst :: (a, b) -> a
fst (x, _) = x
Prelude> fst (0, inf)
```

- Innermost evaluation will fail to terminate here, whereas outermost evaluation produces a result.



Termination II

Innermost evaluation: never terminates

```
inf :: Integer
inf = 1 + inf
fst :: (a, b) -> a
fst (x, _) = x
Prelude> fst (0, inf)
Prelude> fst (0, 1 + inf) -- applying inf
Prelude> fst (0, 1 + 1 + inf) -- applying inf
...
```

Outermost evaluation: terminates in one step

```
inf :: Integer
inf = 1 + inf
fst :: (a, b) -> a
fst (x, _) = x
Prelude> fst (0, inf)
0  -- applying fst
```

Call by name or value?

Call by value

- Also called *eager evaluation*
- Innermost evaluation
- Arguments to functions are always fully evaluated before the function is applied
- Each argument is evaluated exactly once ✓
- Evaluation strategy for most imperative languages

Call by name

- Also called *lazy evaluation*
- Outermost evaluation
- Functions are applied *before* their arguments are evaluated
- Each argument may be evaluated more than once ✗
- Evaluation strategy in Haskell (and others)

Avoiding inefficiences: sharing

- Straightforward implementation of call-by-name can lead to inefficiency in the number of times an argument is evaluated

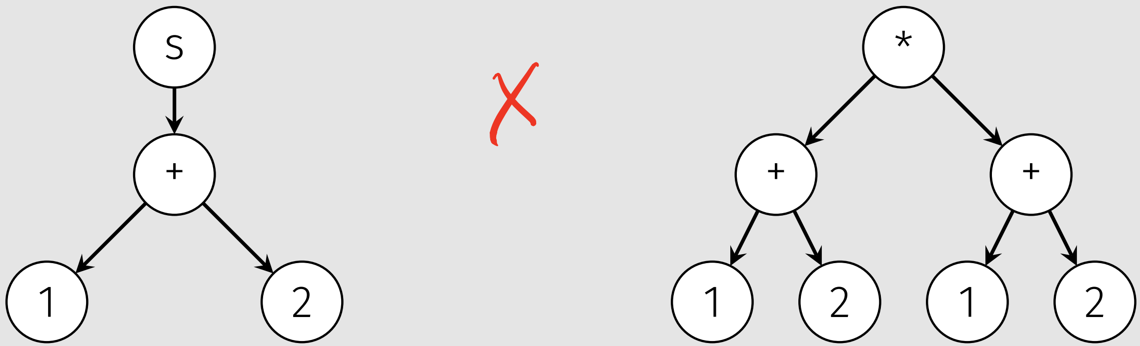
Example

```haskell
square :: Int -> Int
square n = n * n
Prelude> square (1+2)
== (1 + 2) * (1 + 2) -- applying square
== 3 * (1 + 2) -- applying +
== 3 * 3 -- applying +
== 9
```

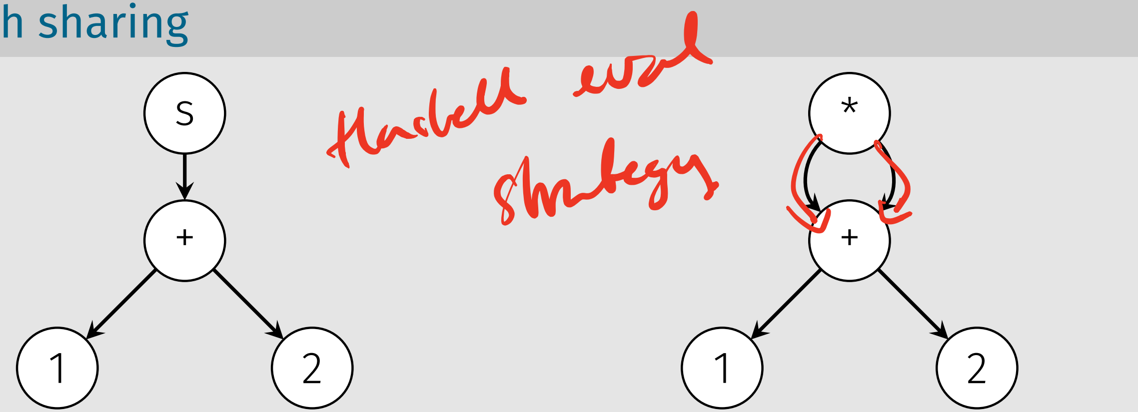
- To avoid this, Haskell implements *sharing* of arguments.
- We can think of this as rewriting the evaluation tree into a graph.

Avoiding inefficiences: sharing

Without sharing



With sharing



Building block summary

- Prerequisites: none
- Content
  - Saw some examples of lazily-evaluated (and infinite) expressions in Haskell
  - Introduced different evaluation strategies for expression graphs: innermost and outermost
  - Defined "call-by-name" and "call-by-value" models of evaluation
  - Discussed termination of the evaluation of expressions
  - Saw how Haskell uses "call-by-value" along with argument sharing (treating the expression tree as a graph)
- Expected learning outcomes
  - student can *describe* difference between call-by-name and call-by-value evaluation schemes.
  - student can *explain* how Haskell uses argument sharing to avoid inefficiency when implementing call-by-value.
- Self-study
  - None

Can we make lazy functions strict? What are some pitfalls.

Controlling evaluation order

How does Haskell evaluate an expression graph?

Definition (Normal form)

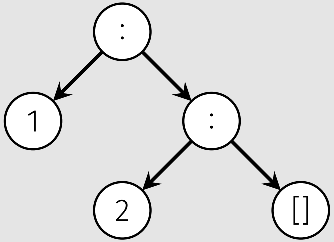
The expression graph contains no redexes, is *finite*, and is *acyclic*.

Data constructors are not reducible, so although they "look" like functions, there is no reduction rule

Example

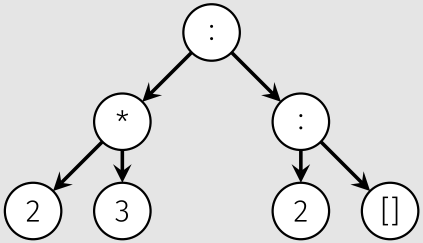
In normal form

`[1, 2] == 1:2:[]`



Not in normal form

`[2 * 3, 2] == (2 * 3):2:[]`



How does Haskell evaluate an expression graph? II

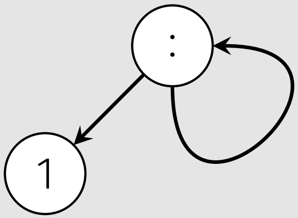
Definition (Weak head normal form (WHNF))

The expression graph is in normal form, *or* the topmost node in a the expression graph is a constructor.

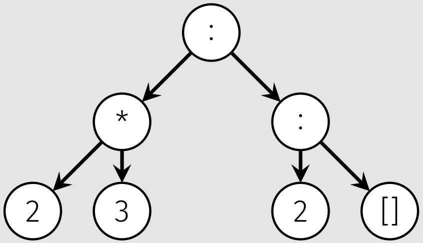
This allows for cycles.

Example

```haskell
ones = 1 : ones
```




```haskell
[2 * 3, 2] == (2 * 3):2:[]
```

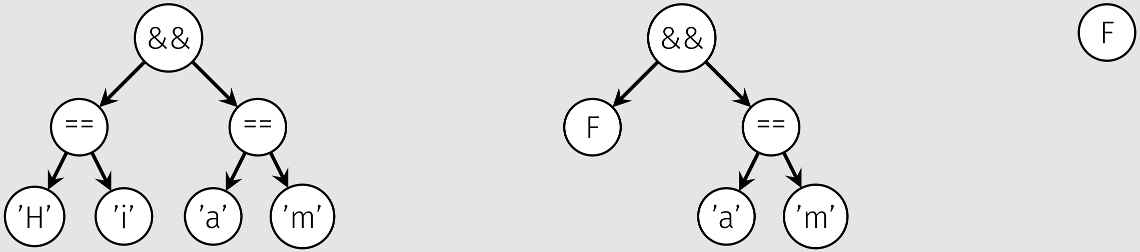


Evaluation rule

- Apply *reduction rules* (functions) *outermost first*
- Evaluate children "left to right"
- Stop when the expression graph is in WHNF
- Function definitions introduce new *reduction rules*

Example

```
('H' == 'i') && ('a' == 'm')
```



Right hand (second) argument is never evaluated. In this way, we get "short circuit" evaluation for free for *all* functions.

Lazy evaluation in strict languages

- All (probably!) languages have one place where they do something akin to lazy evaluation

Boolean expressions

```c
#include <stdlib.h>
int blowup(int arg)
{
   abort();
}
int main(int argc, char **argv)
{
   return (argc < 10) || blowup();
}
```

- Boolean expressions do *short circuit* evaluation
- Avoids evaluating unnecessary expressions
- But not possible when assigning to variables.

Lazy evaluation in strict languages II

- Python generators are lazily evaluated

Infinite generator of integers

```
import itertools
def integers():
    i = 0
    while True:
        yield i # yield control to caller
        i = i+1

for p in itertools.takewhile(lambda x: x < 5, integers()):
    print(p)
0
1
2
3
4
```

- Somewhat painful to work with when combining them

Strict functions

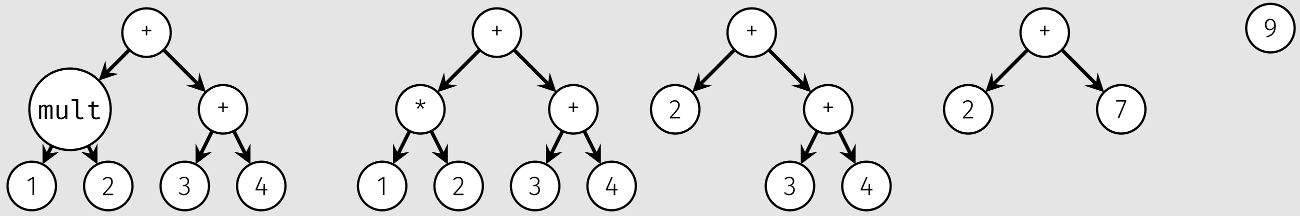
Definition (Strict function)

A function which requires its arguments to be evaluated before being applied.

Even when using outermost evaluation.

- Some functions in Haskell are strict (normally when working with numeric types)

Example



Strict functions: saving space

- Haskell uses lazy evaluation by default
- It also provides a mechanism for *strict* function application, using the operator ($!)

```
($!) :: (a -> b) -> a -> b
f $! x -- evaluate x then apply f
```

- When using ($!), the evaluation of the argument is forced *until* it is in weak head normal form.

Example

```
square $! (1 + 2)
== square $! 3 -- applying +
== square 3 -- applying $!
== 3 * 3 -- applying square
== 9 -- applying *
```

- This allows us to write functions that evaluate as if we had call-by-value semantics, rather than the default call-by-name

Strict functions: saving space II

- Lazy evaluation can require a large amount of space to generate the expression graph

```
sumwith :: Int -> [Int] -> Int
sumwith v [] = v
sumwith v (x:xs) = sumwith (v+x) xs
Prelude> sumwith 0 [1, 2, 3]
== sumwith (0+1) [2, 3]
== sumwith ((0+1)+2) [3]
== sumwith (((0+1)+2)+3) []
== (((0+1)+2)+3)
== ((1+2)+3)
== (3+3)
== 6
```

- This formulation generates an expression graph of size $\mathcal{O}(n)$ in the length of the input list
- In contrast, strict evaluation always evaluates the summation immediately, using constant space.

Saving space III

- This kind of strict evaluation *can* be useful
- `sumwith` is "just" a tail recursive left fold

```
sumwith = foldl (+) 0
```

- For a strict version, which will use less space, we can use `foldl'`

```
import Data.Foldable
sumwith' = foldl' (+) 0
```

- This can have reasonable time saving for large expressions

Example

```
Prelude> foldl (+) 0 [1..10^7]
2 secs
Prelude> foldl' (+) 0 [1..10^7]
0.25 secs
```

- Aside: it is probably a historical accident that `foldl` is not strict (see `http://www.well-typed.com/blog/90/`)

Building block summary

- Prerequisites: none
- Content
    - Introduced the evaluation rules for Haskell expressions
    - Defined terms *normal form* and *weak head normal form*
    - Saw some examples of "lazy" evaluation in strict languages
    - Saw how to define strict functions in Haskell using `($!)`
    - Saw an example where strict evaluation can improve runtime (but note this is not a silver bullet)
- Expected learning outcomes
    - student can *explain* Haskell's evaluation rules for expressions
    - student can provide an *example* of "lazy evaluation" in strict languages
    - student can *write* strict functions in Haskell
- Self-study
    - None